Acta Technica 62 (2017), No. 5B, 735–744

Supply chain enterprises' production decision-making mechanism considering carbon tax

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Abstract. This paper researches the production decision problem of reduction subsidies and reduction cooperation in upstream and downstream enterprises. In this paper, we study optimal output and consider the issues of the level of emission reduction, optimal profit and social welfare in two cases to discuss subsidies for implementation and enterprises' cooperation. The article finally selects some reasonable data to verify the important conclusion.

Key words. carbon tax; social welfare; government regulation of carbon emission; carbon emission reduction rate.

1. Introduction

China is a big manufacturing country, the total carbon emissions in 2011 has exceeded the placecountry-regionUnited States ranked first in the world. Reducing carbon emissions has become a major problem for enterprises to face. Therefore, our country should make scientific and reasonable environmental policy; only in this way can we promote the enterprise effectively. Among them, the government's subsidy and the cooperation between enterprises are two important means to regulate the behavior of enterprises, and their effective implementation must be carried out under certain environmental policy. However, the government's cooperation in reducing emissions can only be encouraged but cannot be enforced, whether cooperating or not depends on the pursuit of profit maximization of the enterprise requirements.

[1]Acknowledgement - This paper is funded by the National Science and Technology Support Program of China (No. 2012BAC20B12).

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2. Formulation of the problem

Early in the last century, Pigou in the "welfare economics" (1920) pointed out that the unit investment marginal private net production will unreasonably exceeds the marginal social net yield, the result is almost in all countries, to impose special taxes for this business[1]. The most obvious form of these incentives and restrictions, of course, is the subsidy and the tax. Bernard and Jose[2] pointed out that the carbon tax is the way to curb carbon emissions by economic incentives.

Some scholars begin to pay attention to the research of enterprise operation decision. Song& Leng[3] study the stochastic demand further with the newsboy model, considering decision problem of optimal order quantity of single cycle enterprises due to three carbon emissions restrictions (mandatory carbon tax and emission reduction, carbon cap and trade), Nie[4] through establishing a carbon emission reduction efficiency model of representative energy enterprises to observe how the input price changes affect production decisions, and analyzed the carbon tax policy of how different effects of carbon emission reduction efficiency. BRUCEA[5] analyzes how uncertainty in the future of environmental taxation will affect the investment of capital, and how these changes will affect the investment in pollution control. Erin and Ekunday[6] studied the research and development level of the enterprise profit maximization under the uncertainty of carbon tax.

We can see from the above literature, previous research on corporate emission reduction incentive problem is confined to the environmental policy research object, limited to a single enterprise or duopoly enterprises. However, researches in the supply chain upstream and downstream enterprises emission reduction subsidies and cooperation are scarce. On the basis of the above documents, this paper studies the policy incentives of the government and the joint emission reduction of the upper and lower reaches enterprises under certain carbon emission tax.

3. Parameter Description

Enterprises face an exogenous carbon emission tax; the tax rate is t, that is, the government levies a price per unit of carbon emissions tax. Companies can reduce carbon emissions by reducing emissions, thereby reducing taxes. Consumers' demand for the product is a linear function, that is $Q = N - p$, (which N is market capacity; P is price of product). Upstream enterprises' emission reduction investment is $\frac{1}{2} \cdot m \cdot \tau^2$ (which m for the reduction of the degree of difficulty, the greater the value of m, the more to invest, emission reduction is more difficult, on the contrary, emission reduction is easier; τas the level of emission reduction, can be seen as a reflection of emission reduction efficiency. If used as a percentage, $\tau = \frac{after\ reduced}{before\ reduced}$ and then if you use the level, $\tau = 1, 2, 3, \cdots, n$. Here setting emission reduction emission reduction level is used to reflect the characteristics of decreasing marginal, i.e. $I'(\tau) = m \cdot \tau > 0, I"(\tau) = m > 0$,) and firms' abatement investment will not change the traditional production cost size, upstream products will according to wholesale prices to downstream retailers.

Assuming the production units of suppliers' products will produce carbon emissions of a unit, that is $e_m = 1$. And the total carbon emissions of enterprises are only positive correlation with the yield and have nothing to do with the unit product carbon emissions. The government will levy a tax on the total carbon emissions from enterprises, and to implement subsidies and incentives for investment enterprises to cooperate with each of the two types of incentives. Assuming the share coefficient of government subsidies for emission reduction investment enterprises is γ.

4. Mathematical model

This paper compares and analyzes the two policies of cooperation between enterprises and government subsidies, so it will produce two kinds of game process. Firstly, for emission reduction subsidies, game process: the first stage ,government set the subsidies allocation ratio asγ, the second stage, upstream manufacturers make sure the emission reduction level τ and the wholesale price level determination of emission reduction$w$, the third stage, downstream retailers to determine the retail price of the product p; secondly, on cooperation of emission reduction, the upstream manufacturers determine cooperation emission reduction levelτ in the first stage, the second stage the downstream retailers determine the price of a product$p$. In the course of two kinds of game, the utility of emission reduction technology spillover is not considered.

4.1. Emission Reduction Subsidies

When the government subsidies for enterprises to reduce emissions, for the upper reaches of the manufacturer, its profit function will consist of three parts: sales revenue, emission reduction investment and carbon tax expenditures. Among them, a part of the amount of investment is subsidized by the government. Therefore, its profit function can be expressed by the following formula.

$$\pi_m = [w - c_m - t \cdot (1 - \tau)] \cdot (N - p) - (1 - \gamma) \cdot \frac{1}{2} \cdot m \cdot \tau^2 \tag{1}$$

The profit function of the downstream retailer is shown in the formula (2):

$$\pi_r = (p - w) \cdot (N - p) \tag{2}$$

The first order derivative of the retailer's profit function is obtained$\frac{d\pi_r}{dp}$=$N - p - (p - w)$. Because of $\frac{d^2\pi_r}{dp^2} < 0$, make$\frac{d\pi_r}{dp} = 0$, we can get $p^* = \frac{N+w}{2}$(3),$Q^* = \frac{N-w}{2}$(4)

Put formula (3) into the equation(1), due to$\frac{\partial^2\pi_m}{\partial w^2} = -1 < 0$, make$\frac{\partial \pi_m}{\partial w} = 0$, then we can get $w^* = \frac{N+c_m+t\cdot(1-\tau)}{2}$. Put formula(5) into the equation (3),(4), and put p^*,Q^* into type (1), therefore, the optimal emission reduction level can be obtained as shown in the formula (6): $\tau^* = \frac{t\cdot(N-c_m)-t^2}{4(1-\gamma)\cdot m-t^2}$(6)

If the emission level τ^* is defined as the ratio of the unit product emission to the original unit product emission, the value range should be satisfied$\tau^* \in [0, 1)$, and

$N - c_m - t > 0$, we can get $m > \frac{t\cdot(N-c_m)}{4(1-\gamma)}$(7).

Due to $\frac{\partial m}{\partial t} > \frac{N-c_m}{4(1-\gamma)} > \frac{N-c_m}{4} > 0and\frac{\partial m}{\partial \gamma} > \frac{t\cdot(N-c_m)}{4} \cdot \frac{1}{(1-\gamma)^2} > \frac{t\cdot(N-c_m)}{4} > 0$, so emission reduction coefficient m and carbon emission tax rate t is positive correlation, and the correlation coefficient is greater than$\frac{N-c_m}{4}$; the rate of emission reduction subsidies is proportional, and the correlation coefficient is greater than$\frac{t\cdot(N-c_m)}{4}$. So the government's carbon tax rate should be satisfied $t < \min(\sqrt{4(1-\gamma)\cdot m}, \frac{4(1-\gamma)\cdot m}{N-c_m})$(8)

In addition, if the enterprise's emission reduction level is determined as a constant, then there is$\gamma < 1 - \frac{t^2}{4m}$, and there is $\frac{\partial \gamma}{\partial t} < -\frac{t}{2m} < 0and\frac{\partial \gamma}{\partial m} < \left(\frac{t}{2m}\right)^2 > 0$. This shows that when enterprises invest in emission reduction and stability, the government of corporate carbon emissions reduction subsidy rate will decrease with the increase of carbon emission tax, and the absolute value of the negative correlation coefficient is greater than$\frac{t}{2m}$; and reduction subsidies rate γwill increase with the increase of reduction coefficient m, and the correlation coefficient should be less than$\left(\frac{t}{2m}\right)^2$.

The optimal level of emission reduction (6) intow^*, p^*, Q^*, we can get: $w^* = \frac{2(1-\gamma)\cdot m\cdot(N+c_m+t)-N\cdot t^2}{4(1-\gamma)\cdot m-t^2}$(9) ,$p^* = \frac{(1-\gamma)\cdot m\cdot(3N+c_m+t)-N\cdot t^2}{4(1-\gamma)\cdot m-t^2}$(10) ,$Q^* = \frac{(1-\gamma)\cdot m\cdot(N-c_m-t)}{4(1-\gamma)\cdot m-t^2}$(11)

Put equation (9),(10),(11) into the manufacturer's profit function, then we get the maximum value to the manufacturer's profits as shown in (12): $\pi_m^* = \frac{m\cdot(N-c_m-t)^2}{[4(1-\gamma)\cdot m-t^2]^2} \cdot \left[2(1-\gamma)^2 \cdot m - \frac{1}{2}t^2\right]$(12)

At this time the maximum value of the retailer's profit is $\pi_r^* = \frac{(1-\gamma)^2\cdot m^2\cdot(N-c_m-t)^2}{[4(1-\gamma)\cdot m-t^2]^2}$(13)

Next, we can get the total social welfare after simplified that $F = (N-c_m)\cdot Q^* - \frac{{Q^*}^2}{2} - \frac{1}{2}m\cdot\tau^2$(14)

Order$\frac{dF}{d\gamma} = 0$, the optimal emission reduction investment sharing coefficient can be determined as shown in the formula (15) $\frac{dF}{d\gamma} = \left[N - c_m - \frac{(1-\gamma)\cdot m\cdot(N-c_m-t)}{4(1-\gamma)\cdot m-t^2} - \frac{4m\cdot(N-c_m-t)}{4(1-\gamma)\cdot m-t^2}\right] \cdot \frac{m\cdot t^2\cdot(N-c_m-t)}{[4(1-\gamma)\cdot m-t^2]^2} = 0$(15)

So the optimal proportion of the government's investment in reducing emissions from the government to the manufacturing enterprises can be calculated: $\gamma^* = \frac{5m\cdot t-(N-c_m)\cdot(m+t^2)}{m\cdot(3N-3c_m+t)}$(16)

Also we can get that $\frac{d\gamma^*}{dt} = \frac{m\cdot\left[c_m\cdot t\cdot(12N+t)+16m\cdot(N-c_m)-6t\cdot(N^2+c_m^2)-N\cdot t^2\right]}{m^2\cdot(3N-3c_m+t)^2}$(17)

Make the function of the tax rate t $\Theta(t) = -(N-c_m)\cdot t^2 - 6(N-c_m)^2\cdot t + 16m\cdot(N-c_m)$, if the function$\Theta(t) > 0$, then$\frac{d\gamma^*}{dt} > 0$, if$\Theta(t) < 0$, then$\frac{d\gamma^*}{dt} < 0$, so the function$\Theta(t)$ and$\frac{d\gamma^*}{dt}$ have the same character. Because of $\Delta = 36(N-c_m)^4 + 64m\cdot(N-c_m)^2 > 0$ and two coefficients$-(N-c_m) < 0$, the curve is concave, and there are two points and that there is a maximum value of the horizontal axis. If the tax as the abscissa and ordinate the thought, and the horizontal curve will have two focust^-, t^+. Make$-(N-c_m)\cdot t^2 - 6(N-c_m)^2\cdot t + 16m\cdot(N-c_m) = 0$, the relationship between$\Theta(t)$ and t is shown in figure 1. This shows that at that time$t \in (0, t^+)$, γ^* improved with the increase of t, but the scale is smaller and smaller; at that time$t \in (t^+, +\infty)$, γ^* is decreased with the increase of t, and scale is more and more large. Therefore, at the time$t = t^+$, the government subsidy coefficient reached the maximum value. Also because at that time$t = 0$, by the formula (16) can be known$\gamma^* = \frac{5m\cdot t-(N-c_m)\cdot\left(m+t^2\right)}{m\cdot(3N-3c_m+t)} = -{}^1\!/_3$, the relationship is shown in figure 2. The two focal points of the curve and the horizontal coordinate are $(t_1, t_2) = \left(\frac{5m-\sqrt{25m^2-4m\cdot(N-c_m)^2}}{2(N-c_m)}, \frac{5m+\sqrt{25m^2-4m\cdot(N-c_m)^2}}{2(N-c_m)}\right)$.

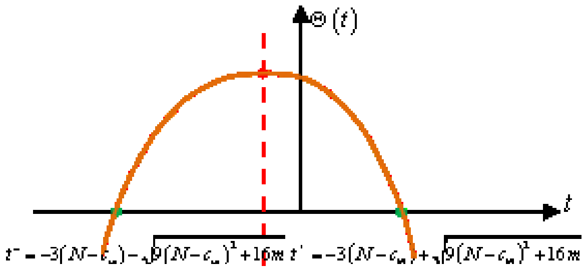

Fig. 1. $\Theta\,(t)$and t's relation

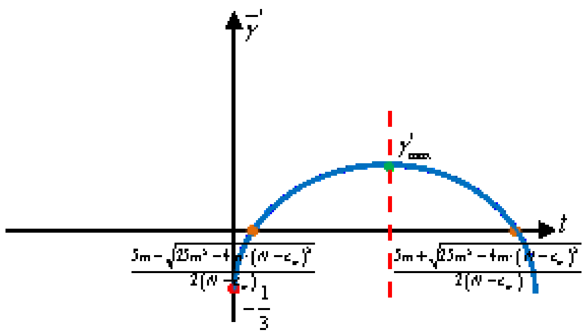

Fig. 2. $\gamma^*$and t's relation

If considering the impact of carbon emissions on the environment, and assume that the damage function of carbon emissions $D\,(Q)$as a linear function of the total output$Q$, use d as the damage coefficient of carbon emissions, then $D\,(Q)$can be expressed by the formula $D\,(Q) = d \cdot (1-\tau) \cdot Q(18)$

Under the condition of maximum value, there are $Q = Q^+, \tau = \tau^+$. The type (6), (11) into the equation (18), available type (19): $D = \frac{4(1-\gamma)\cdot m-t\cdot(N-c_m)}{4(1-\gamma)\cdot m-t^2} \cdot \frac{(1-\gamma)\cdot m\cdot d\cdot(N-c_m-t)}{4(1-\gamma)\cdot m-t^2}$(19). The formula (19) is added to the total social welfare function type (12), and the first derivative of the reduction sharing coefficient is obtained, and make $\frac{\theta F_D}{\theta\gamma} = 0$, we can get:we can get:

$$t^2 \cdot (N - c_m) - \frac{t^2 \cdot (1-\gamma) \cdot m \cdot (N - c_m - t)}{4(1-\gamma) \cdot m - t^2} - \frac{4m \cdot t^2 \cdot (N - c_m - t)}{4(1-\gamma) \cdot m - t^2} - d \cdot (1-\gamma) \cdot [4(1-\gamma) \cdot m - t \cdot (N$$

(20)

From the formula (20) we can see that when the tax rate$t = 0$, there is$4d \cdot m \cdot (1-\gamma)^2 = 0$, therefore, the optimal emission reduction sharing coefficient$\gamma^* = 1$. That is to say, in considering carbon emissions caused by external influences on society, when the government not to impose a carbon tax on carbon emissions enterprises, enterprises will lose power reduction, if the government through cost sharing subsidies for emission reduction enterprises, so the only way is the government will bear all the cost of emission reduction.

Put the government investment proportion γ^* into the optimal reduction type (6), optimal levels for emission reduction τ^*can be expressed as $\tau^* = \frac{t \cdot (3N - 3c_m + t)}{16m + t^2}$ (21) . We can obtain the maximum value $\tau^*_{\max}$ in different levels of carbon emissions reduction of tax t, the method is same to get $\gamma^*_{\max}$ under different carbon emissions tax t, here no unnecessary detail. Put the type(16) into (12) and (13), the maximum available upstream manufacturers and downstream retailer's profit function is respectively shown (22) and (23).

$$\pi^*_m = \frac{2\left[4m \cdot (N - c_m - t) + t^2 \cdot (N - c_m)\right]^2 - \frac{1}{2} t^2 \cdot m \cdot (3N - 3c_m + t)^2}{(16m + t^2)^2}$$

(22)

$$\pi^*_r = \frac{\left[4m \cdot (N - c_m - t) + t^2 \cdot (N - c_m)\right]^2}{(16m + t^2)^2}$$

(23)

4.2. Cooperation and Emission Reduction

This part considers the ways to reduce carbon emissions through cooperation. The total profit of the supply chain is the sum of the profits of the upstream manufacturers and the downstream retailers. That is $\pi_{co} = [p - c_m - t \cdot (1-\tau)] \cdot (N - p) - \frac{1}{2} \cdot m \cdot \tau^2$(24).

According to the inverse solution method, the second order derivative satisfied$\frac{\partial^2 \pi_{co}}{\partial p^2} = -2 < 0$, so the first order partial derivative of the retail price of the product can be obtained that $\frac{\partial \pi_{co}}{\partial p} = (N - p) - [p - c_m - t \cdot (1-\tau)]$. So there is $p^*_{co}$to make the profit of the supply chain to achieve the local optimum, make$\frac{\partial \pi_{co}}{\partial p} = 0$, we can get: $p^*_{co} = \frac{N + [c_m + t \cdot (1-\tau)]}{2}$(25), $Q^*_{co} = \frac{N - [c_m + t \cdot (1-\tau)]}{2}$(26)

Put the type (25), (26) into the equation (24), the profits of the supply chain is available$\pi_{co} = \frac{\{N - [c_m + t \cdot (1-\tau)]\}^2}{4} - \frac{1}{2} \cdot m \cdot \tau^2$. Get the first derivative of the level of emission reduction, obtained $\frac{\partial \pi_{co}}{\partial \tau} = \frac{\{N-[c_m+t\cdot(1-\tau)]\}\cdot t}{2} - m \cdot \tau$, because the second order derivative satisfies $\frac{\partial^2 \pi_{co}}{\partial \tau^2} = t^2 - 2(1-\gamma)\cdot m < 0$, there is optimal emission reduction level τ_{CO}^* making the profit of supply chain get to the local optimal value. Make the first derivative equal to zero, the solution is $\tau_{co}^* = \frac{t\cdot(N-c_m-t)}{2m-t^2}$(27)

Put the type (27) into the equation (25) and (26), we can get $p_{co}^* = \frac{m\cdot(N+c_m+t)-N\cdot t^2}{2m-t^2}$(28), $Q_{co}^* = \frac{m\cdot(N-c_m-t)}{2m-t^2}$(29).

Putp^*, Q^*, τ_{co}^* into the profit function of supply chain, we can get that $\pi_{co}^* = \frac{m^2\cdot(N-c_m-t)^2}{(2m-t^2)^2} - \frac{1}{2}\cdot m \cdot \frac{t^2\cdot(N-c_m-t)^2}{(2m-t^2)^2}$. After simplification, the optimal profit of the supply chain is shown as the formula (30): $\pi_{co}^* = m\cdot\left(m-\frac{1}{2}t^2\right)\cdot\frac{(N-c_m-t)^2}{(2m-t^2)^2}$(30).

4.3. Comparison and Analysis

From the above process of solving the game, in the two cases of government subsidies and cooperation between enterprises, the upper reaches of the manufacturer's optimal emission reduction levels were: $\tau_{nc}^* = \frac{t\cdot(3N-3c_m+t)}{16m+t^2}and\tau_{co}^* = \frac{t\cdot(N-c_m-t)}{2m-t^2}$

The D-value between the two is(31), due to, the character of depends on the function's value . If , then there is, at this point the cooperation that achieve the emission reduction effect is better than the government subsidies, it shows that the government subsidies is better.

If put the tax as the x-coordinate, function as vertical axis, then the curve will have two points of intersection with the horizontal axis. That is, and as a result of the two term, the curve is shown in Figure 3.

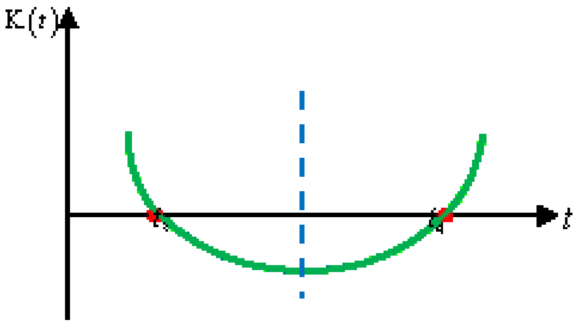

Fig. 3. Schematic diagram of carbon emission tax rate and relationship

Compared with the previous income figure 2,3 can be seen, when other conditions unchanged, at that time$t \in (t_3, t_4)$, $\mathrm{K}(t) < 0$that is$\tau_{co}^* < \tau_{nc}^*$, it shows the government subsidies that get the effect of emission reduction effect is better than the corporate emission reduction. This is mainly due to the moderate carbon tax makes the government is considering to give higher on the emission reduction subsidies, so we can get a higher emission reduction effect; and then at that time$t \in (0, t_3) \vee (t_3, +\infty)$,$\mathrm{K}(t) > 0$ namely$\tau_{co}^* > \tau_{nc}^*$, it shows that the corporate emission reduction is better. The reason for which is that too low or too high carbon tax makes the government not subsidize the reduction of enterprises, while mutual cooperation of the upstream and downstream enterprises will achieve a better reduction effect.

We can get the retailer's optimal optimal product price that:

$$p_{nc}^* = \frac{4m \cdot (3N + c_m + t) + c_m \cdot t^2}{16m + t^2} (32), Q_{nc}^* = \frac{4m \cdot (N - c_m - t) + t^2 \cdot (N - c_m)}{16m + t^2}$$

(33)

Comparison of type(28) and (32), and type (29) and (33) shows that a similar relationship between government subsidies and emission reduction case about the price difference and the yield difference and reduction level difference, that is to look at the tax rate on the value of the function of positive and negative.

The profit value of the entire supply chain can be shown in the form of a government subsidy (34): $\pi_{nc}^* = \pi_{m(nc)}^* + \pi_{r(nc)}^* = \frac{3\left[4m\cdot(N-c_m-t)+t^2\cdot(N-c_m)\right]^2 - \frac{1}{2}t^2\cdot m\cdot(3N-3c_m+t)^2}{(16m+t^2)^2}$ (34)

Comparison of type (30) and (34), considering several limit situations, when government imposed litter on the manufacturing enterprises of the carbon tax, or the government not to impose a carbon tax on it, at that time$t \to 0$, the total profit of the supply chain for government subsidies is $\lim_{t\to 0} \pi_{nc}^* = \frac{3(N-c_m)^2}{16}$; and under cooperation situation is $\lim_{t\to o} \pi_{co}^* = \frac{(N-c_m)^2}{4} > \frac{3(N-c_m)^2}{16}$, so when the government levy the carbon tax rate t is low, the total profit of the supply chain cooperation under the total profit of the supply chain is greater than the case of government subsidies.

5. Numerical Analysis

In order to explain the above problem solving process and its conclusion, also make the calculation simple and convenient, check the hypothesis market capacity$N = 250$, production cost $C_m = 5$, emission reduction factor $m = 100$, carbon emission tax rate $t \in [0.1, 1]$. Table 2 lists when the carbon tax rate is 0.1 step increases gradually, the profit of manufacturer's optimal government subsidies π_m^*, retailers' optimal profitπ_r^*, total profit $\pi_{nc}^*$of the supply chain are corresponded numerical overall supply chain optimal profits $\pi_{nc}^*$and total supply profit in the cooperation (due to limited space, table 2 lists part of data).

6. Conclusions

Studies show that when the government tax rate is relatively low on carbon emissions enterprises, if the government increase the tax rate, the proportion of investment in emission reduction is increasing at the same time; and when the government tax rate is relatively high on the carbon emissions of enterprises, if the government increase the tax rate, it will reduce the contribution to enterprise investment reduction ratio in the same time. In addition, there is a critical carbon tax point which makes the government subsidies and emission reduction cooperation achieve the same reduction effect. Our study contributes to the literature by discussing the relationship between the government tax rate and the proportion of investment in emission reduction. The main findings of this paper have important implications for government subsidy policy and enterprise cooperation on carbon emission reduction.

References

[1] A. C. Pigou: *Welfare economics.* Huaxia express *7* (2007) 177–179.

[2] P. H. Herber, J. T. Raga: *An international carbon tax to combat global warming: An economic and political analysis of the European Union proposal.* The Journal of Economics and Sociology *54* (1995), No. 3, 257–267.

[3] J. Song, M. Leng: *Analysis of the single-period problem under carbon emissions policies.* Applied Acoustics *18* (1985), No. 3, 171–180.

[4] H. Nie: *Optimization choices of carbon tax policies for energy enterprises based on effects of carbon emissions reduction.* Resources science *33* (2011), No. 10.

[5] B. A. Larson, G. B. Frisvold: *Uncertainty over future environmental taxes.* Environmental and Resource Economics *8* (2006), No. 2, 160–180.

[6] E. Baker, E. Shittu: *Profit-maximizing R and D in response to a random carbon tax.* Resource and Energy Economics *28* (2006), No. 2, 160–180.

Received November 16, 2017